DHANAMANJURI UNIVERSITY
DECEMBER-2025

Name of Programme : M.A./M.Sc. Mathematics
Semester : 1st
Paper Code : MAT-503
Paper Title : Topology-I
Full Marks : 80
Pass Marks : 32 **Duration: 3 hours**

The figures in the margin indicate full marks for the questions.

Answers all the questions:

Answer any three from the following questions: **$10 \times 3 = 30$**

1. a) Define:

   i) cofinite topological space,

   ii) discrete metric space,

   iii) limit point of a topological space

   b) Let X be space. Then for $A, B \subseteq X$, show that

   i) $\left(A^0\right)^0 = A^0$ ii) $A \subseteq B \Rightarrow A^0 \subseteq B^0$

   iii) $A^0 \cap B^0 = (A \cap B)^0$ iv) $A^0 \cup B^0 \subseteq (A \cup B)^0$ 3+7=10

2. a) Show that Unitary space C^n is a metric space with the function defined by

   $$d(x,y) = \sqrt{(\xi_1 - \eta_1)^2 + (\xi_2 - \eta_2)^2 + \ldots + (\xi_n - \eta_n)^2}, \quad x, y \in C^n$$

   b) Show that intersection of two topologies $\Im_1$ and $\Im_2$ is again a topology. 6+4=10

3. a) If $(X, \Im)$ is a topological space and $Y \subseteq X$ and $\Im_Y = \{G \cap Y : G \in \Im\}$, then prove that $\Im_Y$ is a topology on Y.

   b) Let $X = \{a,b,c\}$ and $\Im = \{X, \phi, \{a\}, \{b\}, \{a,b\}\}$. Let $A = \{a,c\}$ and $B = \{b\}$ then find $\overline{A}$ and $\overline{B}$. 6+4=10

4. Let X be an infinite set with the cofinite topology and $A \subseteq X$. Prove that if A is infinite, then every point of X is a limit point of A and if A is finite then it has no limit points. 10

Answer any three from the following questions: **$10 \times 3 = 30$**

5. State and prove Urysohn's Lemma. 10

6. a) Define T_0, T_1, T_2, T_3, T_4 spaces.

   b) Let X be a first countable space and Y a space. Show that a function $f: X \to Y$ is continuous iff $x_n \to x$ in X implies $f(x_n) \to f(x)$. 5+5=10

7. Let X and Y be spaces and $f: X \to Y$ a function. Show that the following conditions are equivalent:
   a) f is continuous.
   b) $f^{-1}(F)$ is closed in X for every closed set $F \subseteq Y$
   c) $f(\overline{A}) \subseteq \overline{f(A)}$ for every set $A \subseteq X$
   d) $\overline{f^{-1}(B)} \subseteq f^{-1}(\overline{B})$ for every set $B \subseteq Y$. 10

8. State and prove Tietze Extension theorem. 10

Answer any two from the following questions: **$10 \times 2 = 20$**

9. a) Let $f: X \to Y$ be a continuous map. Show that if X is compact then f(X) is compact.

   b) Show that a cofinite space is compact. 5+5=10

10. a) Show that a compact space has the Bolzano-weierstrass property.

    b) Show that closed subsets of a compact metric space are compact. 5+5=10

11. Let X be a Hausdorff space. Show that

    a) A compact subset of X is closed.

    b) Any two disjoint compact subsets of X have disjoint nbds. 5+5=10

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